MULTICHANNEL IMAGE RESTORATION BASED ON OPTIMIZATION OF THE STRUCTURAL SIMILARITY INDEX

Maja Temerinac-Ott
and
Hans Burkhardt

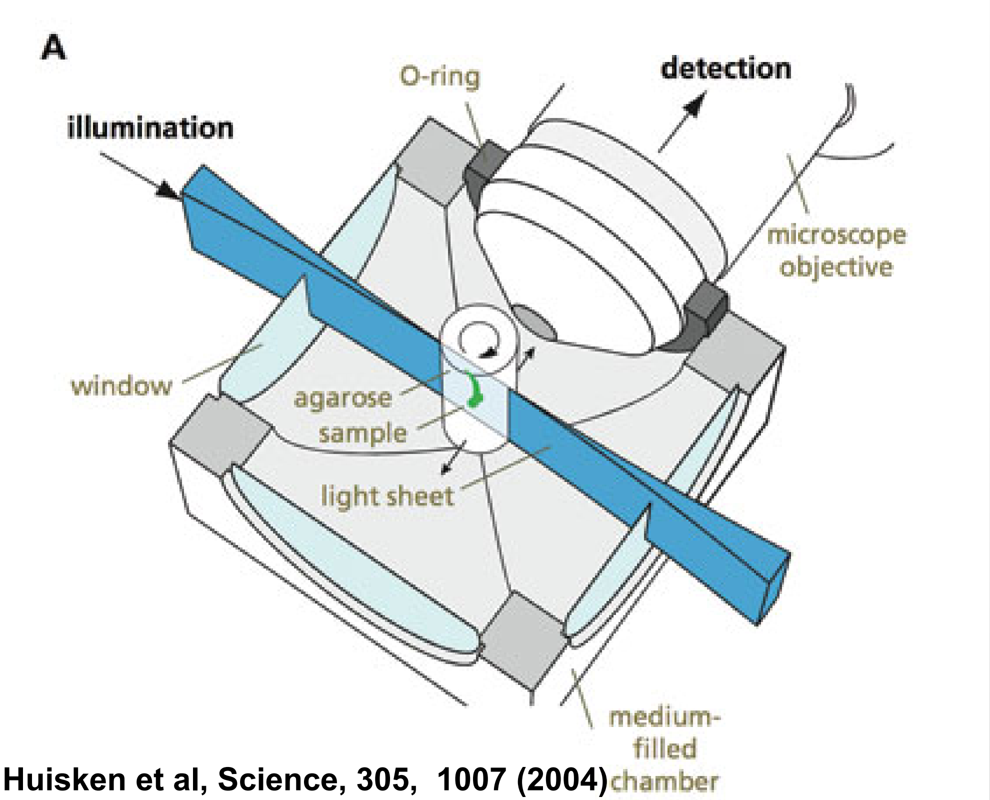
University of Freiburg
Institute for Computer Science
Chair for Pattern Recognition
and Image Processing

OUTLOOK

- Application
  - SPIM
- Mathematical Framework
  - Optimization of SSIM
- Verification on Image data
- Conclusions & Outlook

MOTIVATION

- SPIM (Single Plain Illumination Microscopy) delivers images recorded from different angles

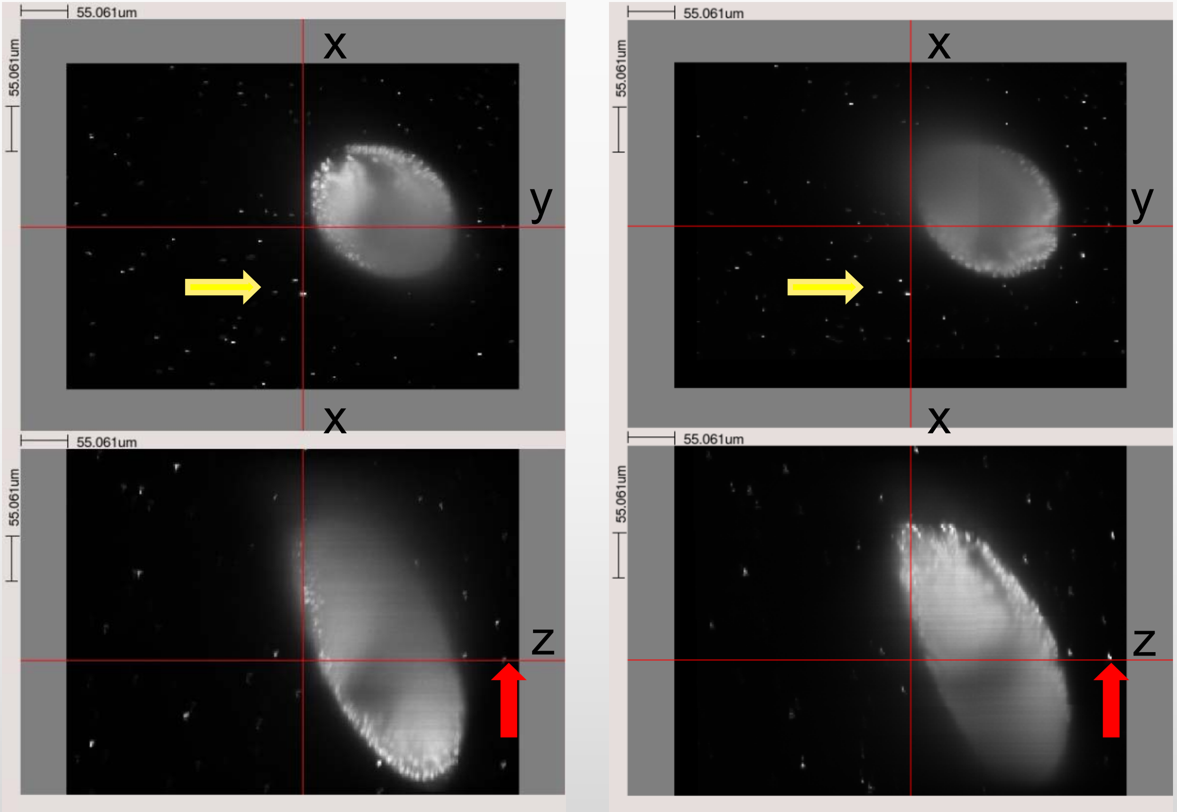


Huisken et al, Science, 305, 1007 (2004)

EXAMPLE SPIM IMAGE

- Drosophila egg recorded from eight angles by Zeiss Jena

left:
Angle 0



right:
Angle 180

- Image size: (1388 x 1040 x 229) voxels, 8 bit (=>300MB)
- Voxel scaling: (0.38µm x 0.38µm x 2µm)

MOTIVATION II

- SPIM images need to be:
  - Registered
  - Fused to one single image
- Due to the PSF of the system and the thickness of the biological sample, the image quality in z-direction is decreasing
- Structural information is missing for almost half of the object when recorded from one angle
- After the fusion more internal structures (e.g. single cells) should be visible!

MOTIVATION III

- Existing SPIM fusion algorithms:
  - Frequency based
  - Deconvolution based on optimization of the MSE

- New fusion algorithm:
  - Deconvolution based on optimization of SSIM

“An optimized system is only as good as the optimization criterion used to design it.”

DEFINITION

- MSE = Mean Squared Error

$$MSE(x,y) = \frac{1}{N}\sum_{i=1}^{N}(x_i - y_i)^2$$

- SSIM[1] = Structural Similarity Index Measure

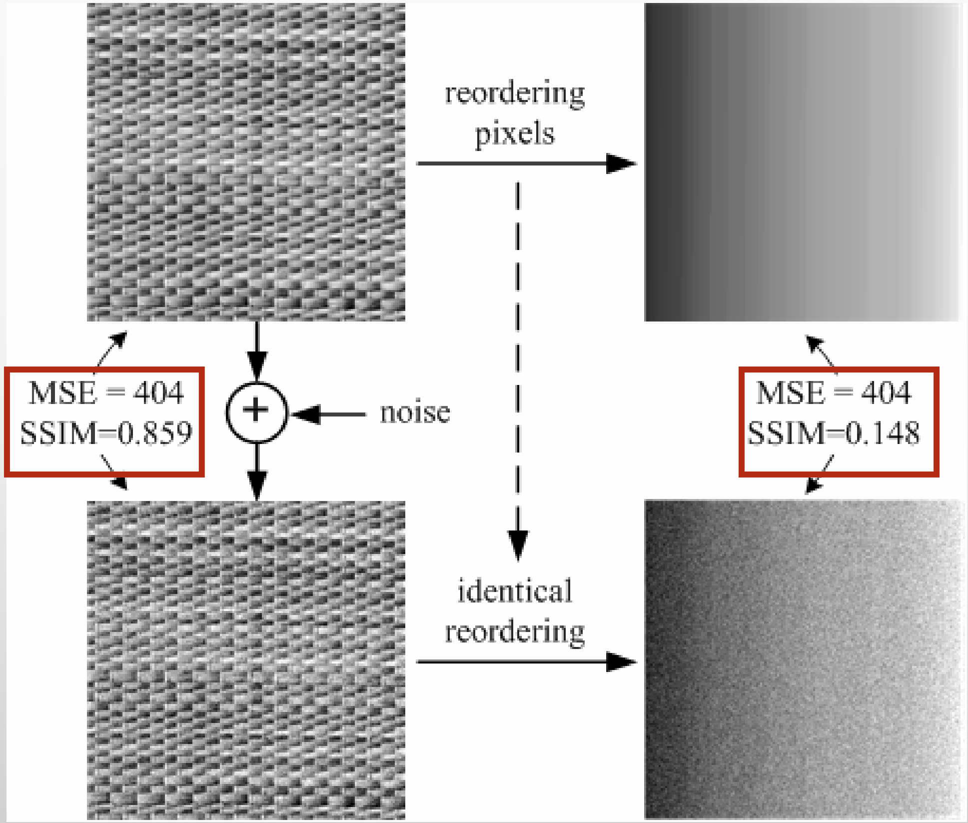
$$SSIM(x,y) = \underbrace{\frac{2\mu_x\mu_y + C_1}{\mu_x^2 + \mu_y^2 + C_1}}_{\textbf{luminance}} \cdot \underbrace{\frac{2\sigma_x\sigma_y + C_2}{\sigma_x^2 + \sigma_y^2 + C_2}}_{\textbf{contrast}} \cdot \underbrace{\frac{\sigma_{xy} + C_3}{\sigma_x\sigma_y + C_3}}_{\textbf{structure}}$$

[1]Z. Wang et al, IEEE TIP, vol. 13, no. 4, pp. 600-612, Apr. 2004

MOTIVATION FOR SSIM OVER MSE

- The structure of the image is important for the visual similarity!!

SSIM High
(noise not visible in the structure)

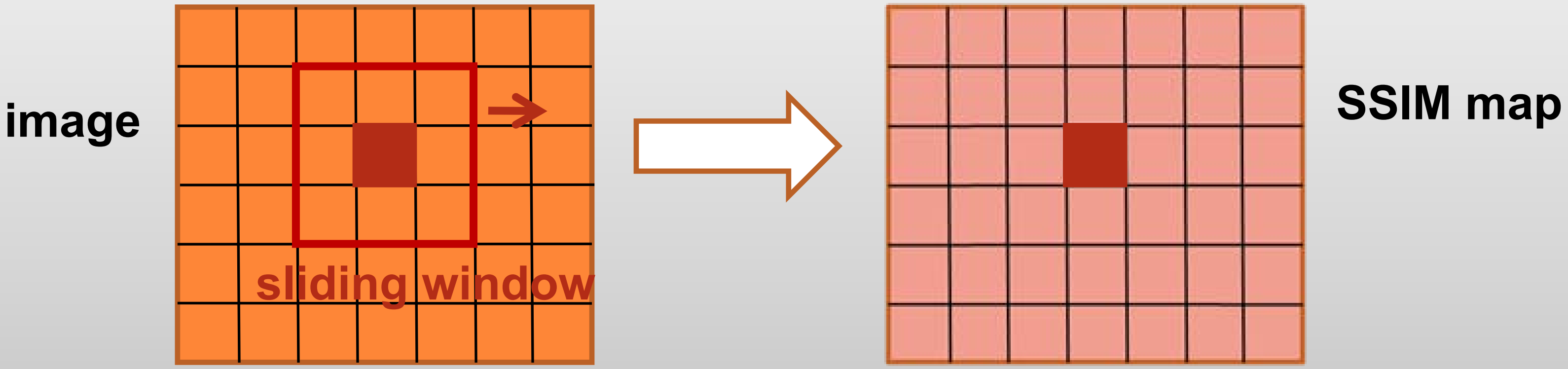


SSIM Low
(noise disturbs the smooth structure)

[2]Z. Wang and A.C. Bovik, IEEE Signal Processing Magazine, vol. 26, no. 1, 2009

COMPUTATION OF SSIM

- SSIM is computed locally within a sliding window that moves pixel by pixel across the image
- For each pixel the result is stored in a SSIM map
- The SSIM value of the whole image can be obtained by averaging the values from the SSIM map



PROBLEM OUTLINE

- The recorded image **y** can be described as a convolution of the original image **x** and the point spread function **h** plus the noise η introduced by the recording system:

$$y = h * x + \eta$$

- GOAL of multi-channel restauration:
  - ***Find the best estimate for*** x ***given the recorded images***
  - ***The quality of the estimate*** $\hat{x}$ ***is computed maximizing the structural similarity index measure***

PROBLEM OUTLINE II

Basic Idea: *Turn* ***non-convex*** *problem into a* ***quasi convex*** *problem*

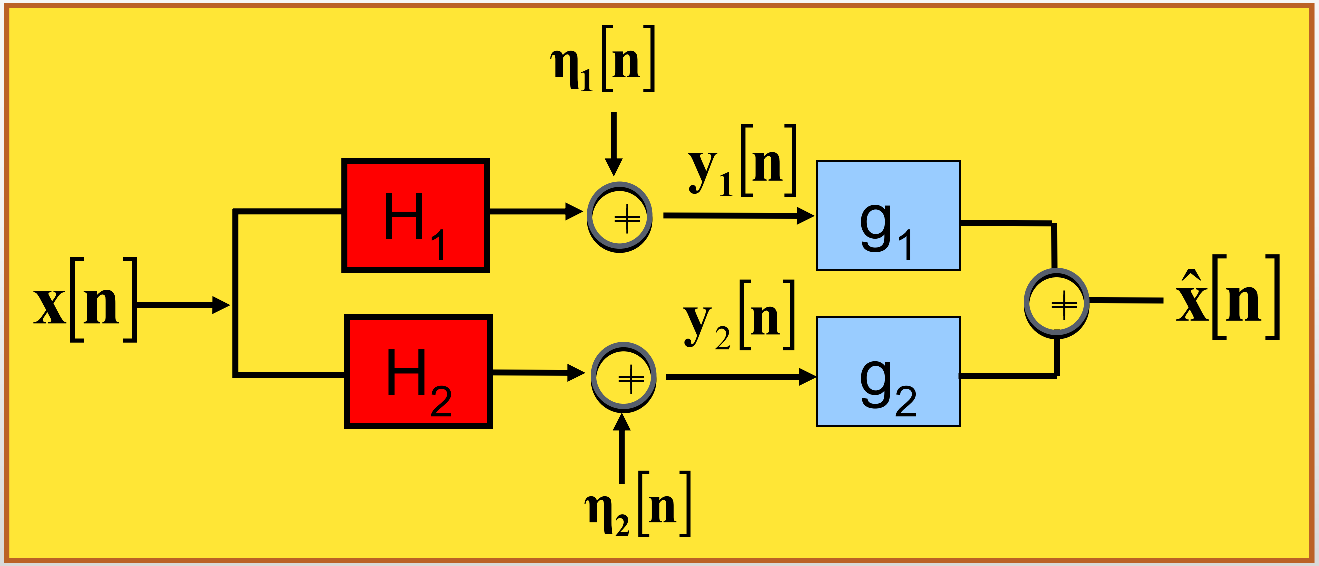
- We use the simplified SSIM[1]:

$$SSIM(x,\hat{x}) = \underbrace{\frac{2\mu_x\mu_{\hat{x}} + C_1}{\mu_x^2 + \mu_{\hat{x}}^2 + C_1}}_{Q_1} \cdot \underbrace{\frac{2\sigma_{x\hat{x}} + C_2}{\sigma_x^2 + \sigma_{\hat{x}}^2 + C_2}}_{Q_2}$$

- It is obtained from the original SSIM index by choosing $C_3 = C_2/2$

EXTENSION OF PREVIOUS WORK

- Restoration problem was solved using SSIM optimization for single channel images[4]
- We extend the solution to multi-channel images:



[4]S.S. Channappayya, IEEE TIP, vol. 17, no. 6, 2008

PROBLEM FORMULATION

- GIVEN:
  - Recorded image $\mathbf{y}_1, \ldots, \mathbf{y}_M$
  - Blurring filters $\mathbf{H}_1, \ldots, \mathbf{H}_M$
  - Probability density function of the noise
- GOAL:
  - Find inverse filters $\mathbf{g}_1, \ldots, \mathbf{g}_M$ such that:

$$\hat{x}[n] = g_1[n] * y_1[n] + \ldots + g_M[n] * y_M[n]$$

  - Maximizing the simplified SSIM

COMPUTING Q_1 AND Q_2

$$SSIM(x,\hat{x}) = \underbrace{\frac{2\mu_x\mu_{\hat{x}}+C_1}{\mu_x^2+\mu_{\hat{x}}^2+C_1}}_{Q_1}\cdot\underbrace{\frac{2\sigma_{x\hat{x}}+C_2}{\sigma_x^2+\sigma_{\hat{x}}^2+C_2}}_{Q_2}$$

- Q_1 is computed by:

$$\mathbf{Q_1} = \frac{2\mu_{\mathbf{x}}E[\sum_{k=1}^{M}\sum_{i=0}^{N-1}\mathbf{g_k}[i]\mathbf{y_k}[n-i]]+C_1}{\mu_{\mathbf{x}}^2+(E[\sum_{k=1}^{M}\sum_{i=0}^{N-1}\mathbf{g_k}[i]\mathbf{y_k}[n-i]])^2+C_1} = \frac{2\mu_{\mathbf{x}}(\mathbf{g_1}^{\mathbf{T}}\mathbf{e}\mu_{\mathbf{y_1}}+\ldots+\mathbf{g_M}^{\mathbf{T}}\mathbf{e}\mu_{\mathbf{y_M}})+C_1}{\mu_{\mathbf{x}}^2+(\mathbf{g_1}^{\mathbf{T}}\mathbf{e}\mu_{\mathbf{y_1}}+\ldots+\mathbf{g_M}^{\mathbf{T}}\mathbf{e}\mu_{\mathbf{y_M}})^2+C_1}$$

- Q_2 is computed by:

$$\mathbf{Q_2} = \frac{2E[(\mathbf{x}[n]-\mu_{\mathbf{x}})(\sum_{k=1}^{M}\sum_{i=0}^{N-1}(\mathbf{g_k}[i]-\mu_{\mathbf{y_k}})]+C_2}{E[(\mathbf{x}[n]-\mu_{\mathbf{x}})^2]+E[(\sum_{k=1}^{M}\sum_{i=0}^{N-1}\mathbf{g_k}[i]-\mu_{\mathbf{y_k}})^2]+C_2} = \frac{2\sum_{i=1}^{M}\mathbf{g_i}^{\mathbf{T}}\mathbf{c_{xy_i}}+C_2}{\sigma_{\mathbf{x}}^2+2\sum_{i=1}^{M}\sum_{j=1}^{M}\mathbf{g_i}^{\mathbf{T}}\mathbf{K_{y_iy_j}}\mathbf{g_j}+C_2}$$

SOLUTION

- Q_1 only depends on $g_i^T e$
- $e = [1,1,\ldots,1]^T$
- Constrain $g_i^T e$ to α_i

$$\mathbf{Q_1} = \frac{2\mu_{\mathbf{x}} E[\sum_{k=1}^{M}\sum_{i=0}^{N-1}\mathbf{g_k}[i]\mathbf{y_k}[n-i]] + C_1}{\mu_{\mathbf{x}}^2 + (E[\sum_{k=1}^{M}\sum_{i=0}^{N-1}\mathbf{g_k}[i]\mathbf{y_k}[n-i]])^2 + C_1} = \frac{2\mu_{\mathbf{x}}(\mathbf{g_1}^{\mathbf{T}}\mathbf{e}\mu_{\mathbf{y_1}} + \ldots + \mathbf{g_M}^{\mathbf{T}}\mathbf{e}\mu_{\mathbf{y_M}}) + C_1}{\mu_{\mathbf{x}}^2 + (\mathbf{g_1}^{\mathbf{T}}\mathbf{e}\mu_{\mathbf{y_1}} + \ldots + \mathbf{g_M}^{\mathbf{T}}\mathbf{e}\mu_{\mathbf{y_M}})^2 + C_1}$$

- The optimization problem is simplified to:

$$\hat{g}(\alpha) = \arg\max_{g \in \Re^{MN}} Q_2 \quad \text{subject to: } g^T e = \alpha$$

- Where **g** is a matrix with the rows being the vectors $\mathbf{g_1}, \ldots, \mathbf{g_M}$

SOLUTION II

- A boundary γ is set to obtain a quasi-complex optimization problem:

min: γ

subject to:

max: $Q_2 \leq \gamma$

subject to:

$$g^T e = \alpha$$

⟺

min: γ

subject to:

min: $f(\gamma) \geq 0$

subject to:

$$g^T e = \alpha$$

LAGRANGE MULTIPLIERS

- The overall problem is now convex and can be solved by applying the Lagrange multipliers

$$\nabla_{g_i}\left(f(\gamma)+\lambda_1\left(g_1^T e-\alpha_1\right)+\ldots+\lambda_M\left(g_M^T e-\alpha_M\right)\right)=0 \qquad \text{(Eq.1)}$$

$$\nabla_{\lambda_i}\left(f(\gamma)+\lambda_1\left(g_1^T e-\alpha_1\right)+\ldots+\lambda_M\left(g_M^T e-\alpha_M\right)\right)=0 \qquad \text{(Eq.2)}$$

- The optimal γ is computed using the bisection method

$\min: \gamma$
subject to:
$\min: f(\gamma) \geq 0$
subject to:
$g^T e = \alpha$

SOLUTION FOR M = 2

- We obtain a system of linear equations (SLE) from the Lagrange multipliers (Eq.1 and Eq.2)

$$\gamma\left(2K_{y_1y_1}g_1 + 2K_{y_1y_2}g_2 - 2c_{xy_1} + \lambda_1 e\right) = 0$$
$$\gamma\left(2K_{y_2y_1}g_1 + 2K_{y_2y_2}g_2 - 2c_{xy_2} + \lambda_2 e\right) = 0$$

from (Eq.1)

$$g_1^T e - \alpha_1 = 0$$
$$g_2^T e - \alpha_2 = 0$$

from (Eq.2)

- Solve SLE!

SOLUTION FOR g_2 USING EQ.1

$$g_2 = g_{2,0} + \lambda_1 g_{2,1} + \lambda_2 g_{2,2}$$

$$g_{2,0} := \frac{1}{\gamma} K^{-1} c$$

$$g_{2,1} := \frac{1}{\gamma} K^{-1} K_{y_2 y_1} K_{y1y1}{}^{-1} e$$

$$g_{2,2} := \frac{1}{2\gamma} K^{-1} e$$

$$K := K_{y_2 y_2} - K_{y_2 y_1} K_{y_1 y_1}^{-1} K_{y_1 y_2}$$

$$c := c_{xy_2} - K_{y_2 y_1} K_{y_1 y_1}^{-1} c_{xy_1}$$

SOLUTION FOR g_1 USING EQ.1

$$g_1 = g_{1,0} + \lambda_1 g_{1,1} + \lambda_2 g_{1,2}$$

$$g_{1,0} := \frac{1}{\gamma} K_{y1y1}^{-1}\left(c_{xy1} - K_{y_1y_2} K^{-1} c\right)$$

$$g_{1,1} := \frac{1}{2\gamma} K_{y_1y_1}^{-1}\left(I - K_{y_1y_2} K^{-1} K_{y2y1} K_{y1y1}^{-1}\right) e$$

$$g_{1,2} := \frac{1}{2\gamma} K_{y1y1}^{-1} K_{y_1y_2} K^{-1} e$$

IMPLEMENTATION

- Filter is implemented pixelwize for a neighborhood of size K×K (here K = 35)
- the covariance $\mathbf{c}_{\mathbf{xy}}$ is estimated using a heuristic technique described by Portilla and Simoncelli[5]
- Each block is made zero-mean before computing the inverse filter; the mean is added back after the computation
- Implementation in Matlab R2009a, for images of size **50×50 pixels** the computation time is **30 sec** on a Intel Core Duo processor with 3 GHz

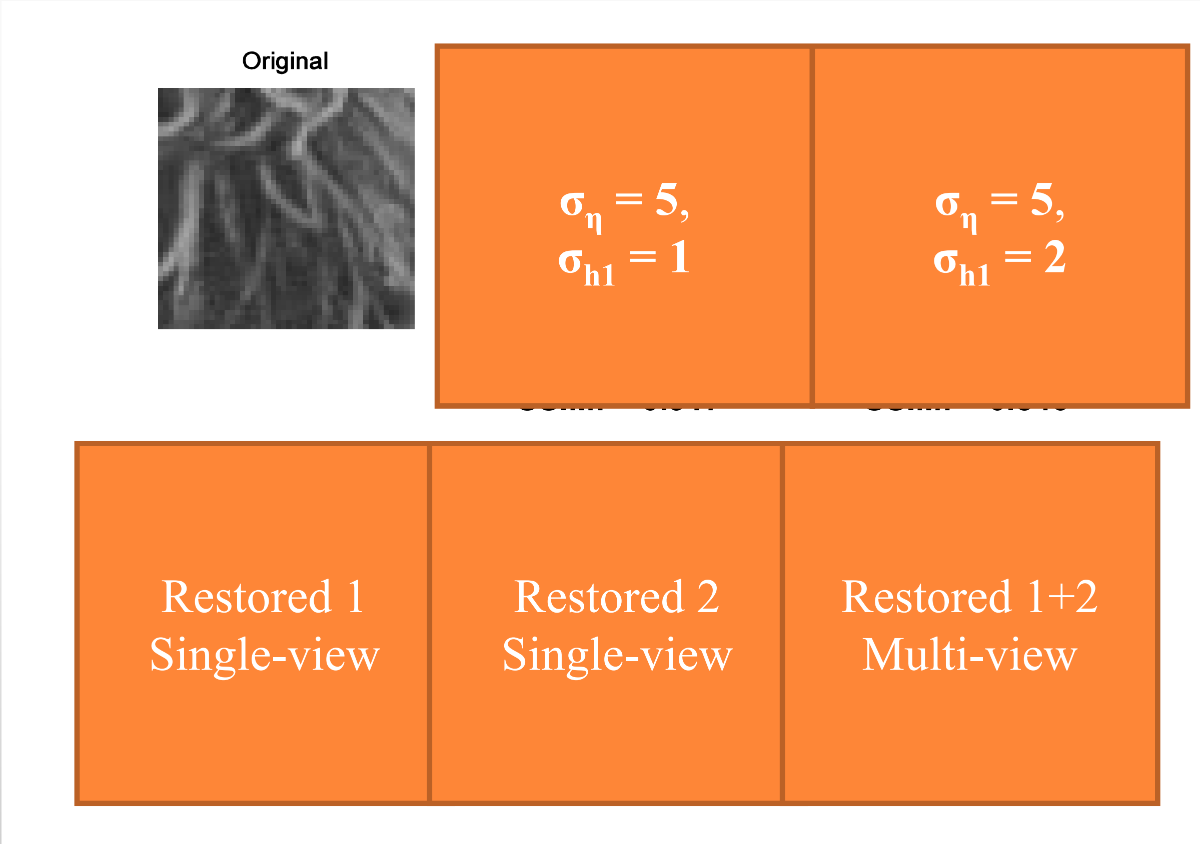
[5] J. Portilla and E. Simoncelli, Proc. of IEEE TIP, vol. 2, pp. 965-968, 2003

RESULTS LENA

- Compare single vs. Multiview reconstruction

The original lena image (top row left) is distorted by **σ_η = 5**, **σ_{h1} = 1** and **σ_{h2} = 2** resulting in the images Distorted 1 and Distorted 2 (top row middle and right).

After SSIM restoration is applied the results are presented for single-channel restoration of Distorted 1 and Distorted 2 (bottom row left and middle) and multi-channel restoration (bottom row right).

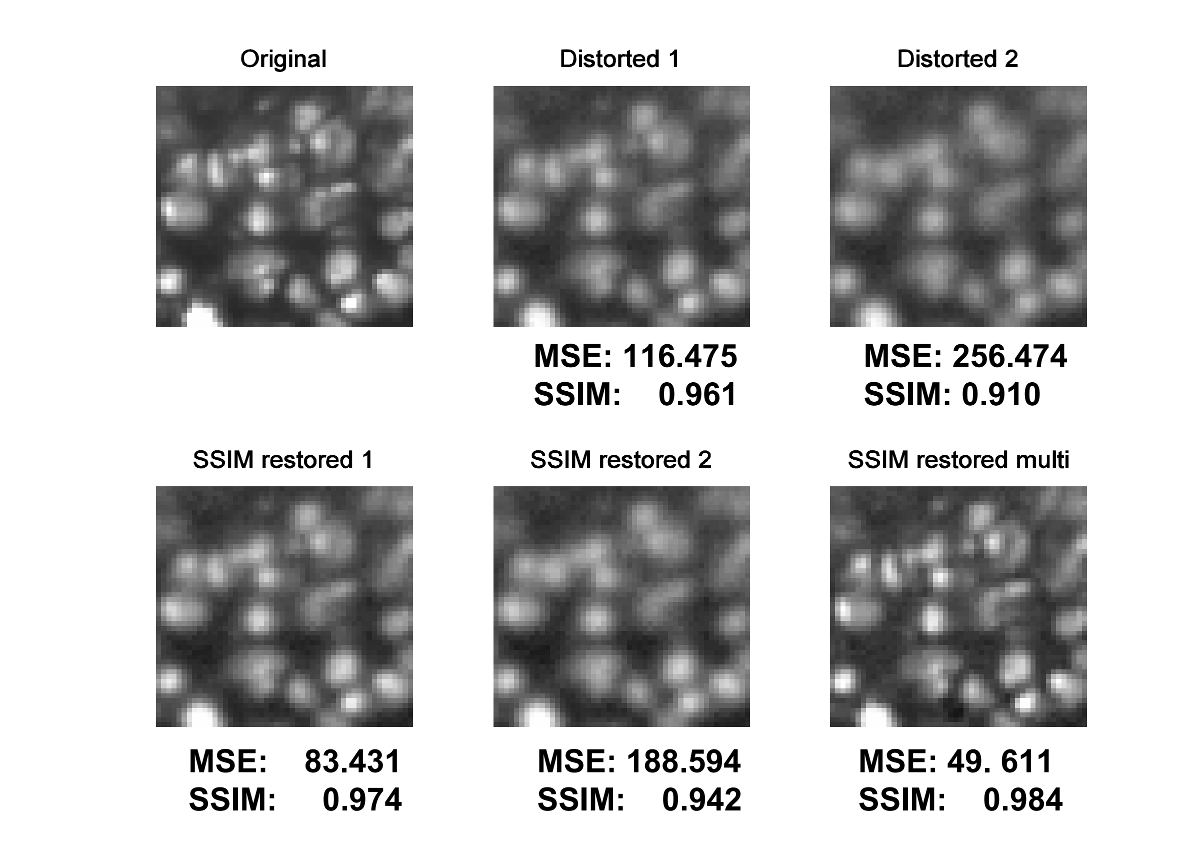


RESULTS DROSOPHILA

- Compare single vs. Multiview reconstruction

The original drosophila image (top row left) is distorted by $\sigma_\eta = 3$, $\sigma_{h1} = 1$ and $\sigma_{h2} = 2$ resulting in the images Distorted 1 and Distorted 2 (top row middle and right).

After SSIM restoration is applied the results are presented for single-channel restoration of Distorted 1 and Distorted 2 (bottom row left and middle) and multi-channel restoration (bottom row right).

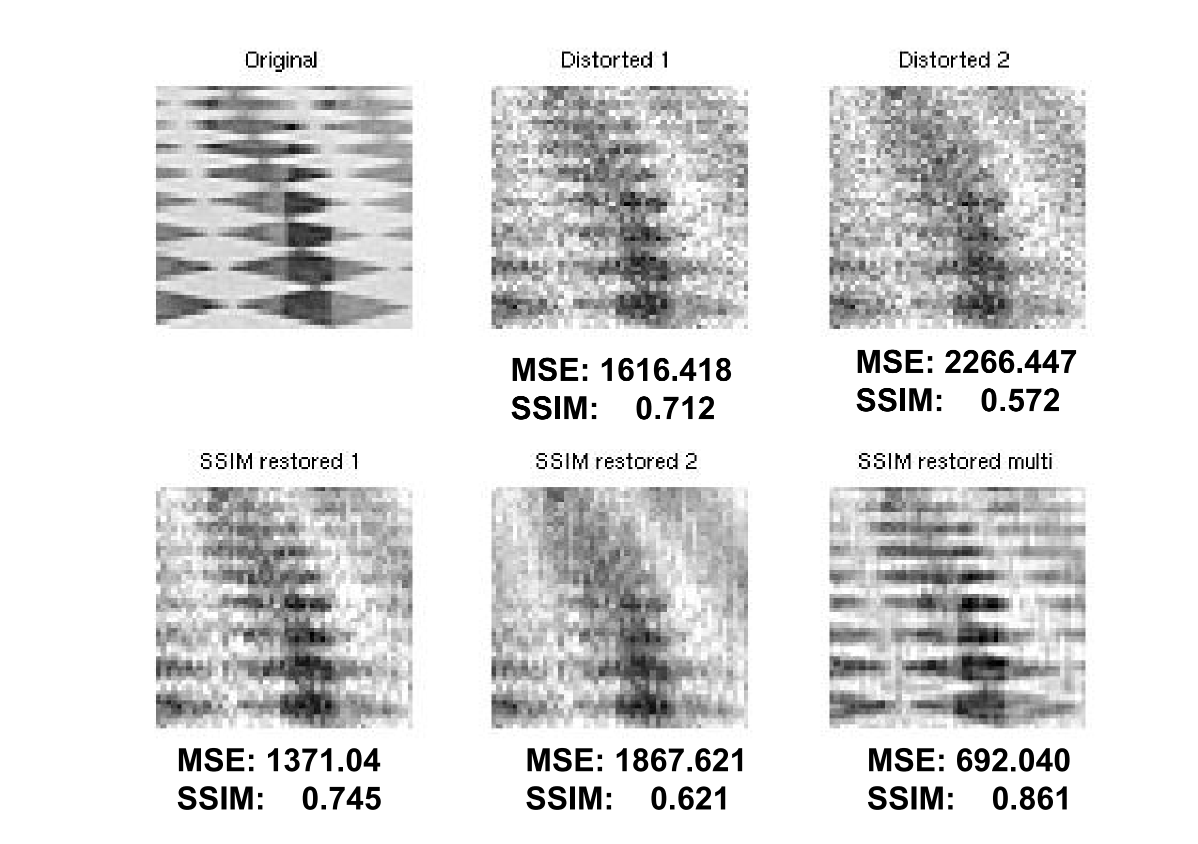


RESULTS CHESSBOARD

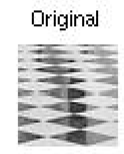
- Compare single vs. Multiview reconstruction

The original checkboard image (top row left) is distorted by $\boldsymbol{\sigma_\eta = 30}$, $\boldsymbol{\sigma_{h1} = 1}$ and $\boldsymbol{\sigma_{h2} = 2}$ resulting in the images Distorted 1 and Distorted 2 (top row middle and right).

After SSIM restoration is applied the results are presented for single-channel restoration of Distorted 1 and Distorted 2 (bottom row left and middle) and multi-channel restoration (bottom row right).



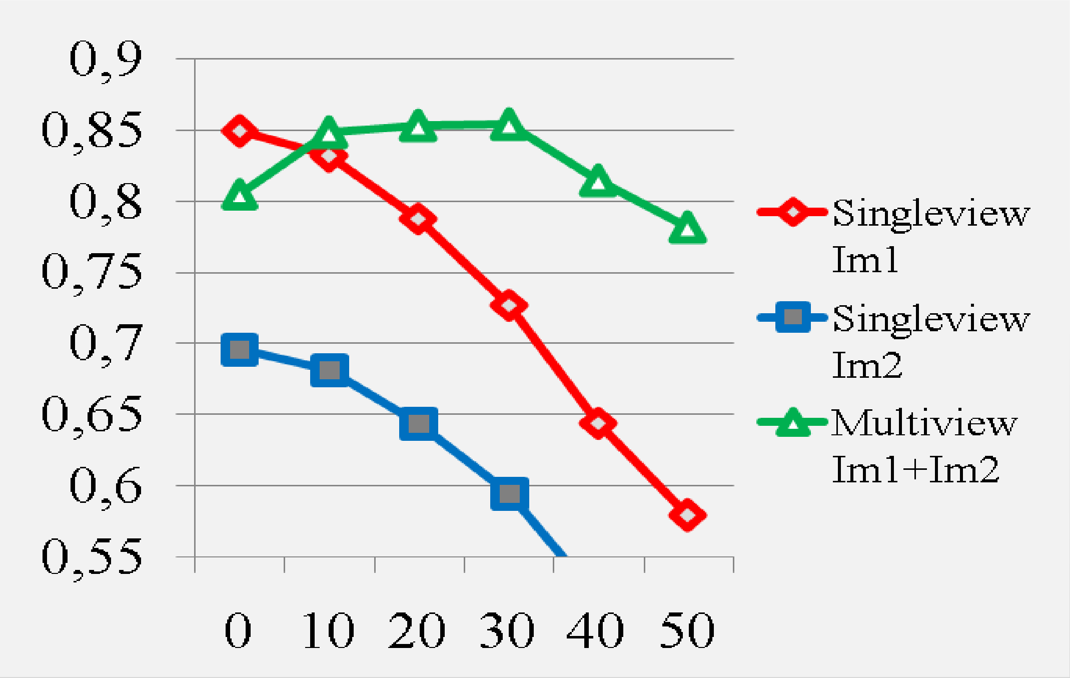
RESULTS: INFLUENCE OF NOISE



- Compare single vs. Multiview reconstruction while alternating the noise on chessboard image

The **influence of noise** (x-axis) on the **SSIM index** (y-axis) is plotted for single-channel restoration (red and blue) and multi-channel restoration (green).

$\boldsymbol{\sigma_{h1} = 1}$ and $\boldsymbol{\sigma_{h2} = 2}$

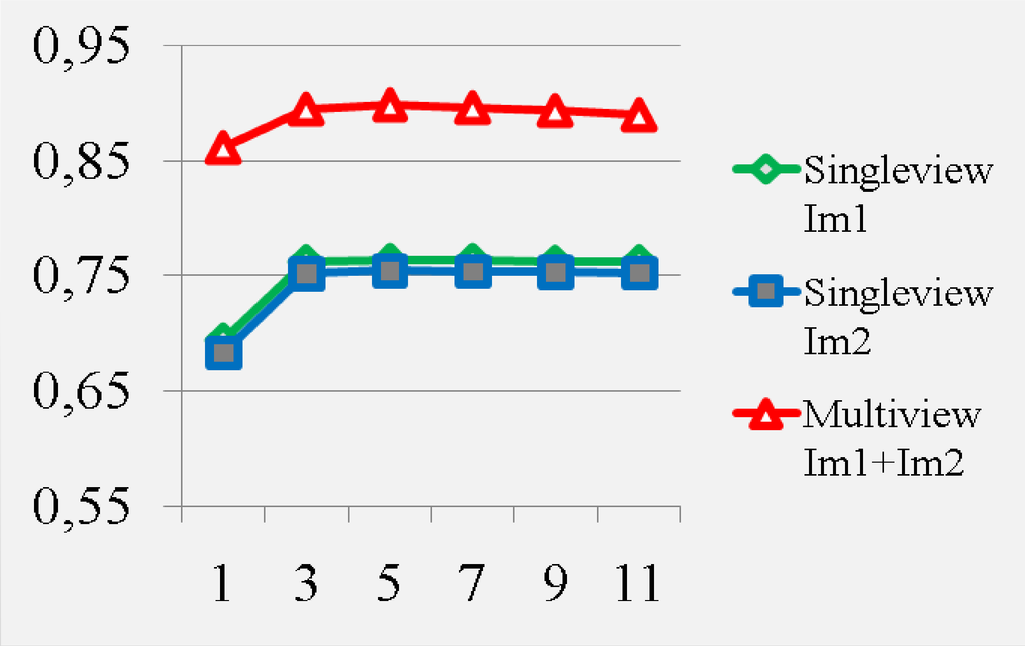


RESULTS: INVERSE FILTER SIZE

- Compare single vs. Multiview reconstruction while alternating the filter size

The **SSIM values** of the single-channel restored image and the multi-channel restored image (y-axis) are plotted against the **filter size** (x-axis).

Here results for the lena image with parameters $\boldsymbol{\sigma_{h1} = 3}$, $\boldsymbol{\sigma_{h2} = 6}$ and $\boldsymbol{\sigma_{\eta} = 15}$ is presented.



CONCLUSIONS

- Multi-channel SSIM image restoration significantly improves the single-channel SSIM restoration.

- Advantages of multi-channel SSIM restoration:
  - very effective if the noise level is high
  - a small filter size is sufficient to achieve optimal reconstruction results
  - local structures are preserved

OUTLOOK

- Disadvantages of the method:
  - high computation time
  - needs an estimate for the original image

- Future research:
  - Compare results with MSE-based methods
  - application to three dimensional images
  - significance of the number of distorted images M

PROJECT TEAM

- Computer Science:
  - Hans Burkhardt
  - Olaf Ronneberger
  - Maja Temerinac-Ott
  - Dominik Rueß
  - Mario Emmenlauer
- Biology I:
  - Wolfgang Driever
  - Alida Filippi
  - Björn Wendik
- ZBSA
  - Roland Nitschke

Thank you for your attention!

UNI FREIBURG

SFB 592
Sonderforschungsbereich 592

bioss
CENTRE FOR BIOLOGICAL SIGNALLING STUDIES

BACKUP

MSE VS SSIM

MSE

- Fast & easy to compute
- Valid distance metric in $\mathbf{R}^N$
- Natural way to define energy of error signal
- Convex, symmetric and differentiable
- Widely used

SSIM

- Models similarity as perceived by human visual system
- Natural image signals are highly structured (strong neighborhood dependencies)
- Symmetric, bounded and has a unique maximum

DEFINTION II

- mean intensity: $$\mu_x = \frac{1}{N}\sum_{i=1}^{N} x_i$$
- Standard deviation: (signal contrast) $$\sigma_x = \left(\frac{1}{N-1}\sum_{i=1}^{N}(x_i - \mu_x)^2\right)^{\frac{1}{2}}$$
- Covariance: $$\sigma_{xy} = \frac{1}{N-1}\sum_{i=1}^{N}(x_i - \mu_x)(y_i - \mu_y)$$
- stabalizing constants: $$C_1, C_2, C_3$$

DISTORTIONS

Structural distortions

- Additive noise and blur
- Lossy compression

→ SSIM describes the visual quality good

Non-structural distortions

- Change of luminance and brightness
- Change of contrast
- Gamma distortion
- Spatial shift

→ Better use Complex Wavelet SSIM[3]

[3] Z. Wang and E. P. Simoncelli, Trans. IEEE ICASSP, vol. 2, pp. 573-576, 2005

PROBLEM FORMULATION II

- The inverse filters $\mathbf{g_1}, \ldots, \mathbf{g_M}$ are found adjointly by optimizing the statistical SSIM index[3]

$$\hat{g} = \arg \max_{g \in \Re^{MN}} StatSSIM\left(x[n], \hat{x}[n]\right)$$

- Where:

$$StatSSIM\left(x[n], \hat{x}[n]\right) = \underbrace{\frac{2\mu_x \mu_{\hat{x}} + C_1}{\mu_x^2 + \mu_{\hat{x}}^2 + C_1}}_{Q_1} \cdot \underbrace{\frac{2\sigma_{x\hat{x}} + C_2}{\sigma_x^2 + \sigma_{\hat{x}}^2 + C_2}}_{Q_2}$$

STATISTICAL SSIM INDEX

$$\mu_x = E\left[x[n]\right]$$

$$\sigma_x^2 = E\left[\left(x[n] - \mu_x\right)^2\right]$$

$$\sigma_{xy} = E\left[\left(x[n] - \mu_x\right)\left(y[n] - \mu_y\right)\right]$$

Backup 6

BISECTION METHOD

- The optimal γ is computed using the bisection method

1. Initialize γ (say γ_0) between 0 and 1.
Set $upLimit = 1$, $lowLimit = \gamma_0$
2. Evaluate the optimal filter.
if $f(\gamma) \geq 0$ **then**
    if $(upLimit - lowLimit) < \epsilon$ **then**
        Optimal γ found.
        Exit.
    else
        Set $\gamma = (upLimit - lowLimit)/2$
        $upLimit = \gamma$.
        Go to step **2**.
    end if
else
    Set $\gamma = (upLimit - lowLimit)/2$
    $lowLimit = \gamma$.
    Go to step **2**.
end if

EXPLANATION

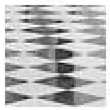
$\mathbf{K_{yy}}$ - covariance matrix
$\mathbf{c_{xy}}$ – cross covaraince vector

- Explicitly Q_2 is:

$$\begin{aligned} \mathbf{Q_2} &= \frac{2E[(\mathbf{x}[n]-\mu_{\mathbf{x}})(\sum_{k=1}^{M}\sum_{i=0}^{N-1}(\mathbf{g_k}[i]-\mu_{\mathbf{y_k}})]+C_2}{E[(\mathbf{x}[n]-\mu_{\mathbf{x}})^2]+E[(\sum_{k=1}^{M}\sum_{i=0}^{N-1}\mathbf{g_k}[i]-\mu_{\mathbf{y_k}})^2]+C_2} \\ &= \frac{2\sum_{i=1}^{M}\mathbf{g_i}^{\mathbf{T}}\mathbf{c_{xy_1}}+C_2}{\sigma_{\mathbf{x}}^2+2\sum_{i=1}^{M}\sum_{j=1}^{M}\mathbf{g_i}^{\mathbf{T}}\mathbf{K_{y_1y_j}}\mathbf{g_j}+C_2} \end{aligned}$$

- And $f(\gamma)$ is:

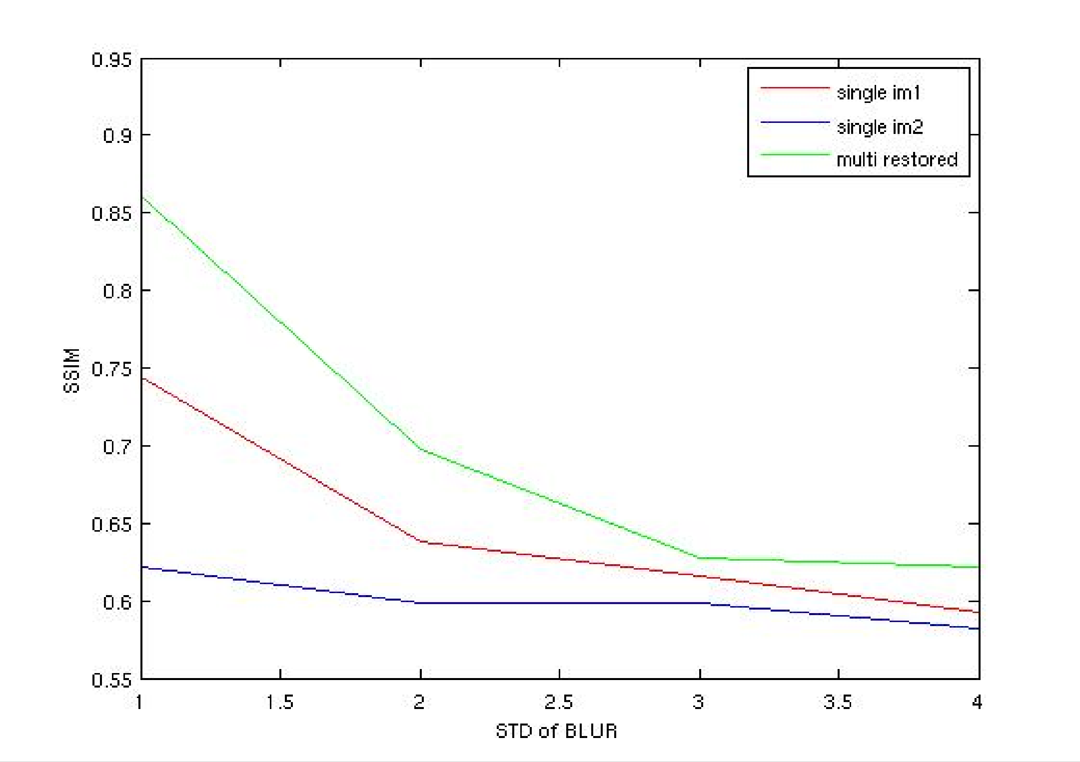
$$\begin{aligned} f(\gamma) = \gamma(\sigma_{\mathbf{x}}^2 + 2\sum_{i=1}^{M}\sum_{j=1}^{M}\mathbf{g_i}^{\mathbf{T}}\mathbf{K_{y_1y_j}}\mathbf{g_j} + C_2) \\ -(2\sum_{i=1}^{M}\mathbf{g_i}^{\mathbf{T}}\mathbf{c_{xy_1}} + C_2) \end{aligned}$$



RESULTS: INFLUENCE OF BLUR STD

- Compare single vs. Multiview reconstruction while alternating the blur STD for the checkboard image

The **SSIM values** of the single-channel restored image and the multi-channel restored image (y-axis) are plotted against the **blur size** $\boldsymbol{\sigma_{h1}}$ (x-axis)



EXAMPLE[2]

a) Reference image

b) Mean contrast stretch
c) Luminance shift

d) Gaussian noise

e) Implusive noise

f) JPEG compression

g) Blurring

h) Zooming out

i) Translation to right

j) Translation to left

k) Rotation counter-clockwise

l) Rotation clockwise

Almost identical MSE!!

[2]Z. Wang and A.C. Bovik, IEEE Signal Processing Magazine, vol. 26, no. 1, 2009

SOLUTION FOR λ USING EQ.2

- We obtain a solution for λ_1 and λ_2 by:

$$\begin{bmatrix} \lambda 1 \\ \lambda 2 \end{bmatrix} = \begin{bmatrix} g_{1,1}^{T} e & g_{1,2}^{T} e \\ g_{2,1}^{T} e & g_{2,2}^{T} e \end{bmatrix}^{-1} \begin{bmatrix} \alpha_1 - g_{1,0}^{T} e \\ \alpha_2 - g_{2,0}^{T} e \end{bmatrix}$$