The B-transform–a new approach to translation invariant feature extraction

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Abstract
In many pattern classification problems, feature extraction must be concentrated on intrinsic shape information, independent of additional unavoidable shifts. In these cases it is advantageous to use translation invariant transforms for getting pure shape properties.

The paper presents a transform (one- and two-dimensional) which is invariant under cyclic permutations of the input pattern. It belongs to a special class of nonlinear transforms with a fast computing graph, very similar to the linear Walsh graph. The transform is denoted as B-transform because only *B*oolean operations are used instead of algebraic ones. It is a generalization of the M-transform, which was defined for binary patterns, to the class of grey scale patterns. Input and output variables are elements of the finite set of dyadic rational numbers II^m, a representation which allows direct computation of data coming from an analog-to-digital converter with fixed resolution.

As a consequence the transform has a very simple and fast hardware realization with logic elements and a fixed word length because the results of all nodal operations are as well elements of the finite set II^m. Using a graph with the same operations in each layer results in a further simplification because only one stage must be realized.

Some further critical features like uniqueness of representation and sensitivity to disturbances are discussed.

Introduction
In many pattern classification problems a translation has no effect on class membership. In such cases one would like to remove this parameter and extract only shape-specific characteristics before evaluating decision functions.

A possible approach is to use translation invariant transforms. A transform T is called translation invariant if

$$T(\mathbf{x}) = T(t_i(\mathbf{x})), \quad t_i \in \mathbb{T} \qquad (1)$$

where x represents a one- or two dimensional finite pattern (vector or matrix) and $\mathbb{T}$ denotes the class of all possible translational operators t_i. Regarding only finite patterns, each translation is used here as cyclic permutation, for example

$$t_i(\mathbf{x}) = \begin{bmatrix} x_{(0+i) \bmod N} \\ x_{(1+i) \bmod N} \\ . \\ . \\ . \\ x_{(N-1+i) \bmod N} \end{bmatrix} \quad (2)$$

Eq. (1) has the effect, that the complete set $\mathbb{C}_i$ of translates of $\mathbf{x}_i$

$$\mathbb{C}_i = \{\mathbf{x}_i | t_j(\mathbf{x}_i) \in \mathbb{C}_i, \forall t_j \in \mathbb{T}\} \quad (3)$$

in the original pattern space is mapped into one point of the feature space. If above that uniqueness of representation of all possible patterns is required, two structural different patterns should map into distinct points in feature space, corresponding to

$$T(\mathbf{x}) \neq T(d(\mathbf{x})), \forall d \in \mathbb{D} \quad (4)$$

where $\mathbb{D}$ denotes the class of all possible deformational operators. Moreover, patterns which are similar, e.g. underlying a little disturbance, should map into points that are close together. This property leads to the requirement that the transform should be continuous with respect to a certain metric.

Under certain restrictions the position invariant property of two-dimensional objects (including translation and rotation) may be reduced to translation invariance. This may be done using an intrinsic equation of a closed curve, for example the curvature as a function of arc length.

The transform discussed here belongs to a general class of translation invariant transforms which can all be computed by a fast algorithm reducing the number of basic operations from N^2 to $N \cdot ld(N)$, whereby N denotes the number of discrete points of a pattern $\mathbf{x}$. The rapid advances in digital technology have stimulated great interest in such transforms and their realization with general or specially dedicated microprocessors.

Definition

A one-dimensional pattern may be represented by a vector $\mathbf{x}$ whose elements x_i all belong to the set Π^m. Π^m denotes the finite set of dyadic rational numbers with m digits, for example the set of non-negative integers less than 2^m in binary representation

$$x_i = x_{i,m-1} \dots x_{i,1} x_{i,0} =$$

$$= \sum_{j=0}^{m-1} x_{i,j} \cdot 2^j, \; x_{i,j} \in \{0,1\} \quad (5)$$

The transform $\tilde{\mathbf{x}}$ of the vector $\mathbf{x}$ is defined as

$$\left.\begin{aligned} x_{2i}^{(r+1)} &= f_1(x_i^{(r)}, x_{i+N/2}^{(r)}) \\ x_{2i+1}^{(r+1)} &= f_2(x_1^{(r)}, x_{i+N/2}^{(r)}) \end{aligned}\right|_{i=0}^{N/2-1} \quad r = 0,1,\dots, n-1 \quad N = 2^n$$

$$\mathbf{x}^{(0)} = \mathbf{x}, \; \tilde{\mathbf{x}} = \mathbf{x}^{(n-1)} \quad (6)$$

with the two commutative operators

$$f_1(x_i,x_k) = x_i \underset{m}{\wedge} x_k = \sum_{j=0}^{m-1} (x_{i,j} \wedge x_{k,j}) \cdot 2^j,$$
$$f_2(x_i,x_k) = x_i \underset{m}{\vee} x_k = \sum_{j=0}^{m-1} (x_{i,j} \vee x_{k,j}) \cdot 2^j \quad (7)$$

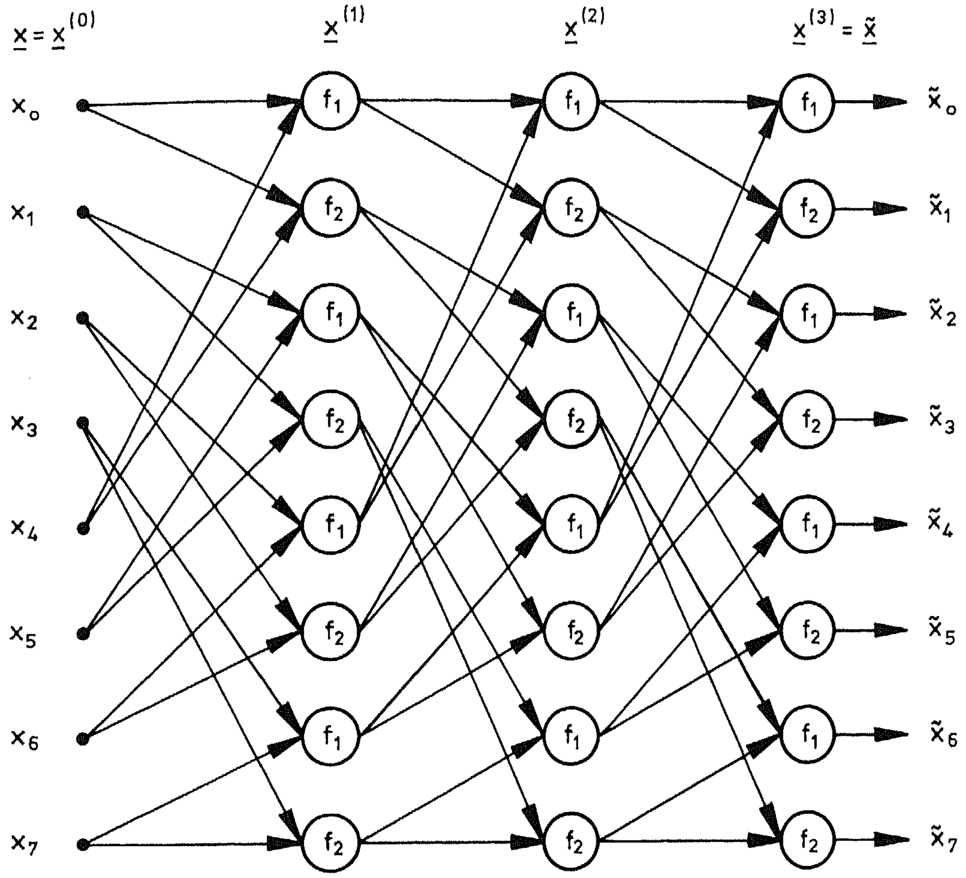
which represent bitwise logical AND bitwise logical OR respectively. Figure 1 shows the corresponding signal flow diagram.

The B-transform belongs to a general class of fast translation invariant transforms [1] with the flow diagram of Fig. 1 and two arbitrary commutative operators f_1 and f_2 in the nodal points. It can be regarded as a generalization of the M-transform [1] for binary patterns to the set of gray scale patterns. Another member of this general class is the R-transform [2] which was published first with the special commutative operators $f_1(x_i,x_k) = x_i + x_k$ and $f_2(x_i,x_k) = |x_i - x_k|$. Without the nonlinear operation $ABS(\cdot)$ in f_2 the R-transform according to the graph in Fig. 1 is identical to the natural ordered fast Walsh transform, following the well-known Cooley-Tukey algorithm.

The transform can be extended to two-dimensional patterns $\mathbf{x}$ of dimension $N \times N$ with invariance under a translation in both directions

$$t_{k,l}(\mathbf{x}) = \{\mathbf{x}_{(i+k) \bmod N, (j+l) \bmod N}\} \quad (8)$$

It may be defined as



$$f_1 \mathrel{\hat{=}} \bigwedge_m \; ; \quad f_2 \mathrel{\hat{=}} \bigvee_m \; ; \quad N = 2^n = 8$$

$$x_i^{(r)} \in \mathbb{I}^m \; ; \quad \begin{array}{l} i = 0, 1, \ldots, N-1 \\ r = 0, 1, \ldots, n-1 \end{array}$$

Fig. 1 Signal flow diagram of the B-transform

$$\left.\begin{array}{ll} x^{(r+1)}_{2i,2j} & = f_1[f_1(x^{(r)}_{i,j}, x^{(r)}_{i+N/2,j}), f_1(x^{(r)}_{i,j+N/2}, x^{(r)}_{i+N/2,j+N/2})] \\ x^{(r+1)}_{2i+1,2j} & = f_1[f_2(. \; , \; . \quad), f_2(\quad . \; , \quad . \quad)] \\ x^{(r+1)}_{2i,2j+1} & = f_2[f_1(\; . \; , \; . \quad), f_1(\quad . \; , \quad . \quad)] \\ x^{(r+1)}_{2i+1,2j+1} & = f_2[f_2(\; . \; , \; . \quad), f_2(\quad . \; , \quad . \quad)] \end{array} \right| {}_{j=0}^{N/2-1} \; \Big| {}_{i=0}^{N/2-1}$$

$$r = 0,1, \ldots, n-1 \; ; \quad N = 2^n$$

$$\mathbf{x}^{(0)} = \mathbf{x}, \; \mathbf{x} = \mathbf{x}^{(n-1)} \qquad (9)$$

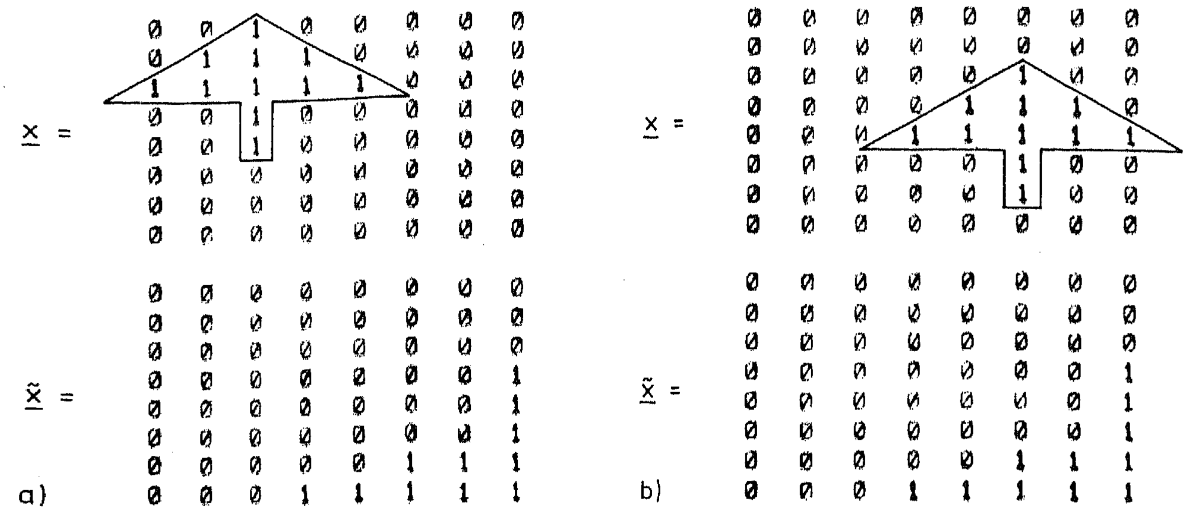


Fig. 2 Example of a binary pattern (a), shifted in both directions (b) and the corresponding unaffected transforms $\tilde{x}$.

with the same operators f_1 and f_2 of Eq. 7. Two examples of the two-dimensional transform (8 × 8) for a binary and a gray scale pattern are given in Figs. 2 and 3. It can be seen that the transformed patterns are unaffected by shift.

Fortran programs for the computation of the one- and two-dimensional transform are given in the Appendix. The algorithms may very easily be modified to other transforms by exchanging the functions f_1 and f_2.

Properties and consequences for implementation

When executed on a digital computer, this transform like the R-transform, is 10-100 times faster than the fast Fourier transform.

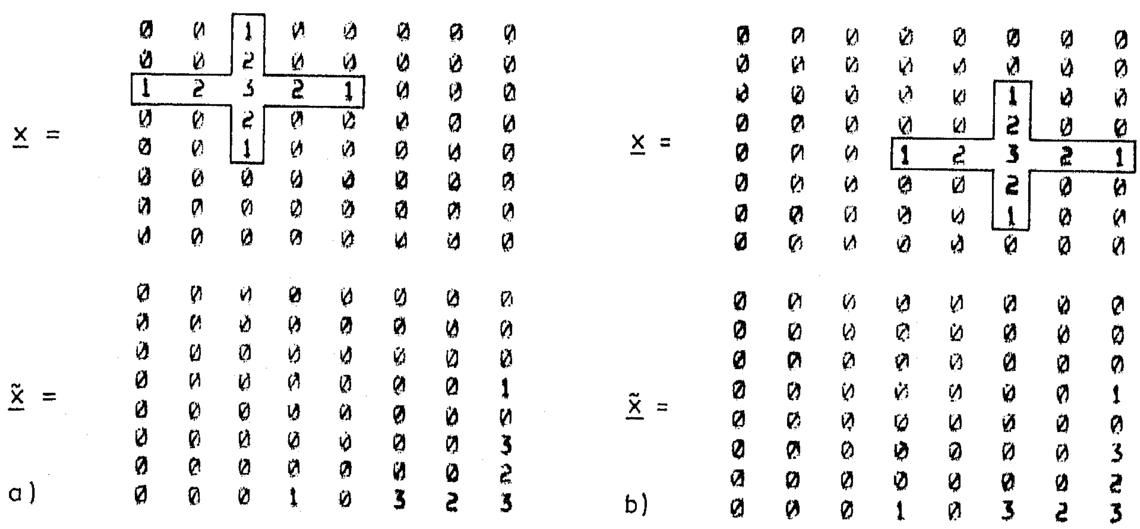


Fig. 3 Example of a gray scale patters (a), shifted in both directions (b) and the corresponding unaffected transforms $\tilde{x}$.

Compared to the R-transform the B-transform with its simple operations in the modal points has further advantages for a special purpose parallel computer with substantial savings in hardware and computation time. Instead of using a full adder in each modal point, the operators f_1 and f_2 may be realized with m parallel gates. One layer of the one- or two-dimensional graph may therefore be computed with the delay of only one logic gate.

All $n = ld(N)$ stages of the signal flow diagram used in Fig. 1 are identical. Hence it is possible to realize a single stage and recirculate data from output to input $ld(N)$ times. As a consequence the computations time for $N = 2^{10} = 1024$ points with 10 stages could be less than 100 ns.

The set II^m is closed under the two operations given in Eq. 7. Hence all elements within the graph including stage 0 (input) and stage $n - 1$ (output) are elements of II^m

$$x_i^{(r)} \in \text{II}^m, \qquad r = 0, \ldots, n-1; \; i = 0, \ldots, N-1 \tag{10}$$

This results in a uniform transform volume. The input data coming from an analog-to-digital converter with a resolution of m bits may be processed or realized throughout the whole signal flow graph with the same constant number of digits or word length.

Applicability

As already mentioned two essential aspects for the applicability of the transform to classification problems are its *sensitivity* and *uniqueness* of representation.

The transform should be immune to little distortions of the pattern which means that it should be continuous with respect to a certain metric. Little modifications in pattern space should result in little modifications in feature space. The following example shows the results of the B-transform used in conjunction with a minimum Euclidean distance classifier. In Fig. 4 three samples of one-dimensional patterns $(x_{0,1}; x_{0,2}; x_{0,3})$ are given with great similarity of pattern 1 and 3. The patterns are distorted by a uniformly distributed additive noise of 2.5, 5, 10, 25 (Fig. 5) and 50%. Tables 1-3 show the respective Euclidean distance matrices in the original pattern space and in the feature spaces of the R- and B-transform. The elements of the distance matrices $\{d_{i,j}\}$ are computed as the distances between the reference patterns and the distorted patterns.

$$d_{i,j} = \|x_{i,0} - x_j\|$$

A classification is possible if all main diagonal elements of the distance matrix are less than the other corresponding column elements. The B-transform shows fairly good results compared to the R- and original space. A good discrimination is given up to a distortion of 25%.

The results of the B-transform, however, are highly dependent on changes in magnification and average value which in contrast to the R-transform may not easily be isolated. It can be shown that the R-transform is continuous with respect to the Euclidean metric, the B-transform, however, not. The results may be improved if cyclic codes (e.g. the Graycode) for the input data are used in which all successive code words differ in exactly one digit.

The B-transform, like the R-transform, is a nonlinear transform which is non-unique with respect to the set of cyclic permutations $\mathbb{C}_i$. The efficiency of the transforms is characterized by the number of distinct elements in feature space, because it determines the maximum number of patterns which can be discriminated. Table 4 shows a comparison of the R- and B-transform (in this case equivalent to the M-transform) for binary output patterns with different dimensions. The table contains corrected values of [1] and shows slight advantages of the R-transform. It can be shown that the general class of fast transforms given in [1] is also invariant to dyadic translation, a translation with componentwise addition modulo 2. A subset of these dyadic translations are the reflection or mirror images, already mentioned in [1] and [2]. In a subsequent paper a complete description of the set of invariants of the general class of transforms given in [1] will be presented.

Conclusions

The paper describes a fast translation invariant transform for processing digitized gray scale patterns with substantial advantages in speed and computational expense over existing transforms. The transform may be very efficiently realized in a special purpose high speed computer. These advantages, however, must be traded against less sensitivity and reduced diversity. Further properties with respect to the various tasks of pattern recognition remain to be explored.

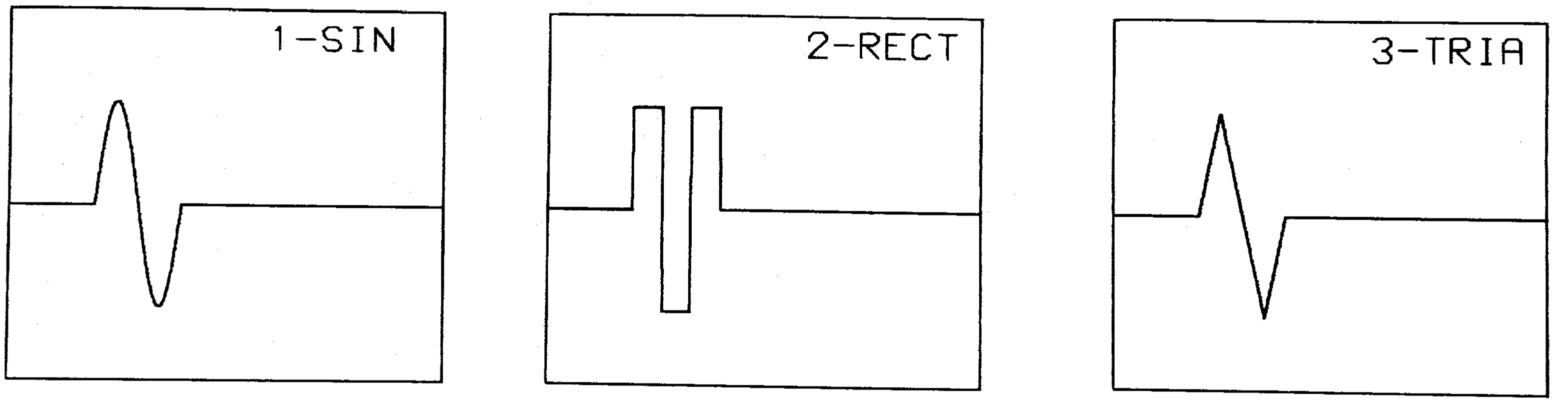


Fig. 4 Samples of one-dimensional patterns ($\mathbf{x}_{1,0}$; $\mathbf{x}_{2,0}$; $\mathbf{x}_{3,0}$).

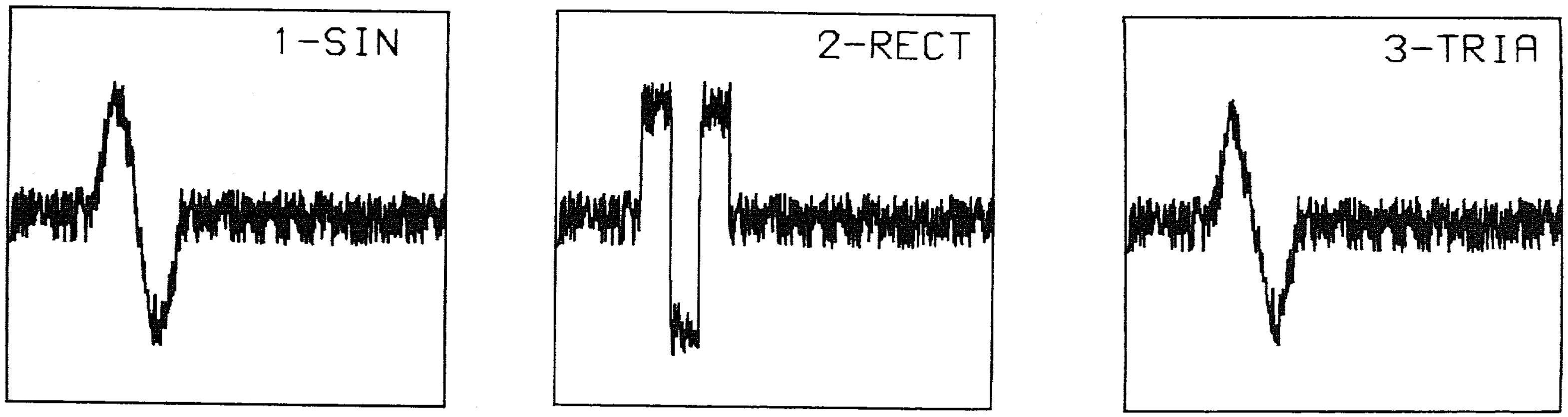


Fig. 5. Patterns of Fig. 4 with a uniformly distributed additive noise of 25%.

TABLE 1. Euclidean distance matrix in original pattern space for the samples given in Fig. 4

	$\underline{x}_1$	$\underline{x}_2$	$\underline{x}_3$
	NOISE LEVEL (IN %): 0.0000000		
$\underline{x}_{1,0}$	0.0000000	12288.21	1508.768
$\underline{x}_{2,0}$	12288.21	0.0000000	11591.38
$\underline{x}_{3,0}$	1508.768	11591.38	0.0000000
	NOISE LEVEL (IN %): 2.500000		
	328.1263	12290.05	1514.361
	12295.12	328.1263	11594.79
	1573.152	11597.24	328.1263
	NOISE LEVEL (IN %): 5.000000		
	656.2527	12300.65	1589.192
	12310.78	656.2527	11607.48
	1699.579	11612.39	656.2527
	NOISE LEVEL (IN %): 10.00000		
	1312.505	12348.01	1906.833
	12368.18	1312.505	11660.57
	2088.563	11670.33	1312.505
	NOISE LEVEL (IN %): 25.00000		
	3281.263	12694.24	3483.600
	12743.23	3281.263	12035.04
	3735.060	12058.67	3281.263
	NOISE LEVEL (IN %): 50.00000		
	6562.526	13885.99	6597.569
	13975.44	6562.526	13298.78
	6867.193	13341.51	6562.526

TABLE 2. Euclidean distance matrix in the feature space of the R-transform for the samples given in Fig. 4

NOISE LEVEL (IN %): 0.0000000		
0.0000000	118574.5	31927.54
118574.5	0.0000000	129287.8
31927.54	129287.8	0.0000000
NOISE LEVEL (IN %): 2.500000		
6021.048	118070.3	31529.01
117184.1	7380.914	127038.9
33402.50	129239.2	6405.528
NOISE LEVEL (IN %): 5.000000		
11933.66	118021.5	32503.93
114753.6	14358.59	124653.6
35557.22	129608.8	12675.71
NOISE LEVEL (IN %): 10.00000		
24370.39	119093.9	36745.31
109668.2	25891.88	120105.4
42180.14	131409.9	24076.82
NOISE LEVEL (IN %): 25.00000		
58463.97	129781.2	56462.18
100620.6	54154.29	109060.3
70760.98	143437.4	55420.48
NOISE LEVEL (IN %): 50.00000		
110558.8	162640.0	97191.26
106851.5	98612.27	103750.2
124398.8	177882.5	107133.5

TABLE 3. Euclidean distance matrix in the feature space of the B-transform for the samples given in Fig. 4 (amplitude of the undistorted patterns in the range $706 < x_{0,i} < 2706$).

NOISE LEVEL (IN %): 0.0000000

0.0000000	13791.51	6751.234
13791.51	0.0000000	14109.11
6751.234	14109.11	0.0000000

NOISE LEVEL (IN %): 2.500000

1050.314	13798.10	6996.008
13622.83	726.3532	14059.08
6892.680	14175.12	2680.296

NOISE LEVEL (IN %): 5.000000

1792.381	13943.04	6601.430
13429.43	1583.919	13728.20
7065.085	14334.14	2418.254

NOISE LEVEL (IN %): 10.00000

3071.077	14324.95	7350.007
13266.62	2707.043	13707.30
7388.342	14837.32	4020.082

NOISE LEVEL (IN %): 25.00000

8192.201	16313.35	9009.083
14119.50	7780.765	13574.42
10548.68	17083.18	8117.683

NOISE LEVEL (IN %): 50.00000

24592.11	28620.72	23852.43
23040.09	24973.27	22560.06
25705.33	29580.13	24935.78

TABLE 4 A comparison for binary patterns

Dimension of the pattern $N = 2^n$	Number of distinct patterns in feature space RT	BT(MT)
4	6	6
8	21	20
16	225	168

APPENDIX I. Fortran program of the one-dimensional B-transform

```
C ONE-DIMENSIONAL B-TRANSFORM
C N=2**M NUMBER OF DATA POINTS
C INPUT-VECTOR IS DESTROYED
C INPUT AND OUTPUT IN IXT(N,1)
C DECLARATION IN MAIN-PROGR.:
C         DIMENSION IX(ND),IXT(ND,2)
C         EQUIVALENCE (IX(1),IXT(1,1))
C
  SUBROUTINE BT(N,ND,IXT)
  DIMENSION IXT(ND,1)
  INTEGER F1,F2,X,Y
  F1(X,Y)=X.AND.Y
  F2(X,Y)=X.OR.Y
C
  NH=N/2
  M=NINT(LOG(FLOAT(N))/LOG(2.))
  IRP1=1
C
  DO 110 I=1,M
  IR=IRP1
  IRP1=IRP1.XOR.3
  DO 100 J=1,NH
  J2=2*J
  JNH=J+NH
  IXT(J2-1,IRP1)=F1(IXT(J,IR),IXT(JNH,IR))
  IXT(J2,IRP1)=F2(IXT(J,IR),IXT(JNH,IR))
100CONTINUE
110CONTINUE
  IF(IRP1.EQ.1)GOTO 200
  DO 150 J=1,N
150IXT(J,1)=IXT(J,2)
200RETURN
  END
```

APPENDIX II. Fortran program of the two-dimensional B-transform

```
C TWO-DIMENSIONAL B-TRANSFORM
C N=2**M NUMBER OF DATA POINTS
C INPUT MATRIX IS DESTROYED
C INPUT AND OUTPUT IN XS(ND,ND,1)
C DECLARATION IN MAIN PROGRAM:
C         EQUIVALENCE (X(1,1),XS(1,1,1))
C         DIMENSION X(ND,ND),XS(ND,ND,2)
C
  SUBROUTINE BT2D(N,ND,XS)
  INTEGER XS,F1,F2,A,B
  DIMENSION XS(ND,ND,1)
  F1(A,B)=A.AND.B
  F2(A,B)=A.OR.B
  M=LD(N)
  NH=N/2
  LP=1
  DO 120 L=1,M
  LL=LP
  LP=LP.XOR.3
  DO 110 J=1,NH
  J2=2*J
  JNH=J+NH
  J2M1=J2-1
  DO 100 K=1,NH
  K2=2*K
  KNH=K+NH
  K2M1=K2-1
  XS(J2M1,K2M1,LP)=F1(F1(XS(J,K,LL),XS(JNH,K,LL)),
  1 F1(XS(J,KNH,LL),XS(JNH,KNH,LL)))
  XS(J2,K2M1,LP)=F1(F2(XS(J,K,LL),XS(JNH,K,LL)),
  1 F2(XS(J,KNH,LL),XS(JNH,KNH,LL)))
  XS(J2M1,K2,LP)=F2(F1(XS(J,K,LL),XS(JNH,K,LL)),
  1 F1(XS(J,KNH,LL),XS(JNH,KNH,LL)))
  XS(J2,K2,LP)=F2(F2(XS(J,K,LL),XS(JNH,K,LL)),
  1 F2(XS(J,KNH,LL),XS(JNH,KNH,LL)))
100CONTINUE
110CONTINUE
120CONTINUE
  IF(LP.EQ.1) GOTO 200
  DO 150 J=1,N
  DO 150 K=1,N
150XS(J,K,1)=XS(J,K,2)
200RETURN
  END
```

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2. REITBOECH, H. and BRODY, T.P. "A transformation with invariance under cyclic permutation for applications in pattern recognition", *Information and Control,* 15, 130-154 (1969).

List of symbols

$\mathbf{x}$	original pattern
$\tilde{\mathbf{x}}$	transformed pattern
$N = 2^n$	dimension of $\mathbf{x}$
m	number of digits of x_i
Π^m	finite set of dyadic rational numbers
$\mathbb{C}_i$	complete set of cyclic permutations of $\mathbf{x}_i$
t_i	translation operator (cyclic permutation)
$\mathbb{T}$	the class of all possible translational operators
$\mathbb{D}$	the class of all possible deformational operators

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