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Übung zur Vorlesung
Algorithmen zur digitalen Bildverarbeitung I

Assignment 7: Pseudoinverse

Datum

Exercise 1:

You are given the following measurements (x_i, y_i):

x_i	-2	-1	0	1	2
y_i	4	2	1	2	3

Estimate the parabola:

$$p(x) = c_0 + c_1 x + c_2 x^2$$

which fits those measurements, by minimizing the quadratic error:

$$\sum_i (p(x_i) - y_i)^2 = \min$$

1. Construct a linear system of equations of the following form: $\mathbf{A}\vec{c} - \vec{y} = \vec{r}$, so that $\vec{c}$ contains the parameters of the parabola, $\mathbf{A}$, $\vec{y}$ the observed measurements, and $\vec{r}$ expresses the error vector which will be minimized.
2. Derive the Gaussian (normal) set of equations $\mathbf{A}^T\mathbf{A}\vec{c} - \mathbf{A}^T\vec{y} = 0$ of the system $\mathbf{A}\vec{c} - \vec{y} = \vec{r}$, by exploiting the requirement of minimizing $\|\vec{r}\|^2$.
3. Estimate the parameters of the parabola $\vec{c}$ by means of the pseudoinverse.
4. Determine the error vector $\vec{r}$.

Exercise 2:

Show by using the definition of the Pseudoinverse matrix, that the matrices: AA^+ and A^+A define orthogonal projections. Give a geometric interpretation of the projections that these matrices define.

Exercise 3:

Consider a matrix of dimensions 3×3: $A = \begin{bmatrix} A' & 0 \\ 0 & 0 \end{bmatrix}$. Suppose A' is a regular 2×2 matrix. Estimate the vector spaces $\mathcal{R}(A)$ and $\mathcal{N}(A)$ and describe all mappings conducted by the matrices A and A^+.

Definition:

Consider $A \in \mathbb{R}^{m\times n}$. The Singular Value Decomposition (SVD) of A is given by the following equation:

$$A = U\Sigma V^T,$$

where $U \in \mathbb{R}^{m\times m}$ and $V \in \mathbb{R}^{n\times n}$ orthogonal matrices, and $\Sigma = (\sigma_i \delta_{ij}) \in \mathbb{R}^{m\times n}$ a diagonal matrix.

Theorem:

Every matrix $A \in \mathbb{R}^{m\times n}$ possesses a Singular Value Decomposition.

Exercise 4:

Calculate the Pseudoinverse of a matrix A, which possesses a Singular Value Decomposition, with Σ matrix defined as:

$$\Sigma = \begin{bmatrix} a & 0 & 0 & 0 \\ 0 & b & 0 & 0 \\ 0 & 0 & c & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$