ALBERT-LUDWIGS-UNIVERSITÄT FREIBURG
INSTITUT FÜR INFORMATIK
Lehrstuhl für Mustererkennung und Bildverarbeitung
Prof. Dr.-Ing. Hans Burkhardt

Georges-Köhler-Allee
Geb. 052, Zi 01-029
D-79110 Freiburg
Tel. 0761 - 203 - 8260

Übung zur Vorlesung
Algorithmen zur digitalen Bildverarbeitung I

Blatt7: Pseudoinverse, Fourier transformation

Datum: 11. June 2009

Aufgabe 1:

The pseudoinverse matrix $\mathbf{A}^+$ of a matrix $\mathbf{A} \in \mathbb{C}^{m \times n}$ is defined by the following axioms:

$$\mathbf{A}\mathbf{A}^+\mathbf{A} = \mathbf{A}$$
$$\mathbf{A}^+\mathbf{A}\mathbf{A}^+ = \mathbf{A}^+$$
$$(A\mathbf{A}^+)^* = \mathbf{A}\mathbf{A}^+$$
$$(\mathbf{A}^+\mathbf{A})^* = \mathbf{A}^+\mathbf{A}$$

Given a system of equations

$$\mathbf{A}\mathbf{v} = \mathbf{b} \tag{1}$$

with $\mathbf{v} \in \mathbb{C}^n, \mathbf{b} \in \mathbb{C}^m$, the pseudoinverse matrix $\mathbf{A}^+$ gives the optimal solution

$$\mathbf{v} = \mathbf{A}^+\mathbf{b} \tag{2}$$

a) Show that if the matrix $\mathbf{A}$ is regular $\Rightarrow \mathbf{A}^+ = \mathbf{A}^{-1}$

b) If (1) has more than one solution, then (2) gives the solution with minimum norm $||\mathbf{v}||$

c) If (1) has no solution, then (2) gives the vector $\mathbf{v}$ with $||\mathbf{A}\mathbf{v} - \mathbf{b}|| \overset{!}{=}$ min

Aufgabe 2:

The convolution of two functions $x(t), y(t)$ is defined by

$$(x * y)(t) := \int_{-\infty}^{\infty} x(t-\tau)y(\tau)d\tau \tag{3}$$

The convolution can also be efficiently performed in Fourier domain.

a) Express the convolution $(x * y)(t)$ in terms of the Fourier transformed $\tilde{x}(f), \tilde{y}(f)$

b) Write down the expressinn $(\widetilde{x * y})(t)$ using the Fourier transformed $\tilde{x}(f), \tilde{y}(f)$